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Bound States of the particles with Identical Charge in Magnetic Field

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Abstract

In this paper we consider bound states and resonances of the particles with identical charge in the presence of the strong magnetic field.

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In this paper we consider bound states and resonances of particles with identical charge in the presence of the strong magnetic field. It is known that in the presence of the strong magnetic field electron movement in Coulomb potential can be considered as one-dimensional.On the other hand, it is known that the term $-\alpha^2/(2mr^2)$ provide an attraction even for the positron. Thus, in one-dimensional case this attraction leads to the existence of the bound states of the same charge (e.g. e^+p,$lepton-lepton$,$antilepton-antilepton$ atoms), because in one-dimensional case any attraction is enough for bound states formation. Thus, we prove that in the strong magnetic field bound states of the same charge exist. Below we calculate energy levels of these bound states.

Dirac equation for an electron in case of attractive potential has been derived in [1][2] where, however, relativistic term (the second term in the formula (2) below) has been neglected.In [3] this term has been taken into account. However in [3] repulsive Coulomb potential has not been considered. Analoguosly the Dirac equation for positron in the repulsive Coulomb potential with the presence of magnetic field (e^+p-atoms) has the following form:

$$(\frac{-1}{2m}(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}+\frac{1}{r^2}\frac{d}{d\phi^2}+\frac{1}{r}\frac{d^2}{dz^2}-\gamma^2r^2+2i\gamma\frac{d}{d\phi})+V(r,z))\psi(x)=E_{eff}\psi(x) \tag{1}$$

where $E_{eff}=\frac{E^2-m^2}{2m}-\frac{eH}{2m}$,

$$V(r,z)=\frac{E}{m}\frac{Z\alpha}{\sqrt{r^2+z^2}}-\frac{Z^2\alpha^2}{2m}\frac{1}{r^2+z^2}, \tag{2}$$

$0<E<m$ Z-charge of nuclei. Of course,in principle it is possible to solve this equation numerically without any assumption.

In accordance with [1][2] if magnetic field is strong (i.e. $\sqrt{\frac{1}{eH}}<<a_0=$

$\frac{1}{mZ\alpha}$,where a_0 - is Borh radius) transverse motion of electrons is defined only by magnetic field and we find the solution in the following form:

$$\psi(r,\phi,z) = \frac{1}{\sqrt{2\pi}} R_{00}(\rho)\chi(z) \tag{3}$$

where $R_{00}(\rho) = \exp(-\frac{\rho}{2})$ is the function of the ground state $n = 0, l = s = 0$, $\rho = \gamma r^2$ ($\gamma = \frac{eH}{2}$)whereas $\chi(z)$ must be found below as a solution to one-dimensional Schredinger equation.Substituting (2)(3) in (1) and multiplying by $\frac{1}{\sqrt{2\pi}} R_{00}$ and integrating over d^2r we obtain:

$$(\frac{d^2}{2mdz^2} + (E_{eff} - \frac{\gamma}{2m})V(z))\psi(x) = 0 \tag{4}$$

where

$$V(z) = \frac{E}{m}\alpha\sqrt{\gamma}\int_0^\infty \frac{d\rho\exp(-\rho)}{\sqrt{r^2+z^2}} - \frac{\alpha^2\gamma}{2m}\int_0^\infty \frac{d\rho\exp(-\rho)}{r^2+z^2} \tag{5}$$

The behaviour of V(z) at different γ is shown on Fig.1. Thus, we have one-dimensional task with attraction at sufficiently small z which guarantees the existence of the bound states and resonances.In [1][2] ρ has been neglected in denominators in formula for $V(z)$ above. We however do not neglect it, because at small z it is not correct and besides the presence of ρ regularized the behaviour of the potential $V(z)$ at small z.If we take into account formfactor of proton we obtain more smooth behaviour of potential at small z.

We solve this one dimensional Schredinger equation (4) numerically. Our numerical results for energy of the ground state versus H at fixed charge of nuclei Z is shown on the Fig.2.

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Figures Caption

Fig.1 The behaviour of V(z) at different γ.(available after request)

Fig.2 Energy of the ground state versus H at fixed Z(available after request)

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